Board Approved August 2017

DEPARTMENT: Mathematics COURSE: Multivariable Calculus

Week	Marking Period 1	Week	Marking Period 3
1	Integration Techniques	21	Functions of Several Variables
2	Integration Techniques	22	Functions of Several Variables
3	Integration Techniques	23	Functions of Several Variables
4	Parametric Equations & Polar Coordinates	24	Functions of Several Variables
5	Parametric Equations & Polar Coordinates	25	Functions of Several Variables
6	Parametric Equations & Polar Coordinates	26	Multiple Integration
7	Parametric Equations & Polar Coordinates	27	Multiple Integration
8	Vectors & The Geometry of Space	28	Multiple Integration
9	Vectors & The Geometry of Space	29	Multiple Integration
10		30	

Week	Marking Period 2	Week	Marking Period 4
11	Vectors & The Geometry of Space	31	Multiple Integration
12	Vectors & The Geometry of Space	32	Multiple Integration
13	Vector-Valued Functions	33	Multiple Integration
14	Vector-Valued Functions	34	Vector Analysis
15	Vector-Valued Functions	35	Vector Analysis
16	Vector-Valued Functions	36	Vector Analysis
17	Functions of Several Variables	37	Vector Analysis
18	Functions of Several Variables	38	Vector Analysis
19	Functions of Several Variables	39	Vector Analysis
20		40	

DEPARTMENT: Mathematics COURSE: Multivariable Calculus

Time Frame	**Block: 5 Days**

Topic

INTEGRATION TECHNIQUES

Essential Questions

1. What are differential equations? How is integration related to differentiation?
2. How is integration used in chemistry, physics, and space?
3. How can integration be used to characterize a child's ability to memorize?

Enduring Understandings

1. Know the relationship between integration and differentiation.
2. Use specific integration techniques when appropriate – integration by parts, partial fractions, trigonometric substitution, completing the square, and trigonometric powers.
3. The units of a particular solution or computation and how it relates to the question.

Alignment to NJSLS

SSE.A.1a, SSE.A.1b, SSE.A.2, CED.A.2, CED.A.4, REI.D.10, REI.D.11, IF.B.4, IF.B.5, IF.B.6, IF.C.6, IF.C.7, MG.A.3, 8.1.12.A.4

Key Concepts and Skills

INTEGRATION BY PARTS

- Find an antiderivative using integration by parts. Recognize when tabular integration is applied.

INTEGRATION BY PARTIAL FRACTIONS

- Understand the concept of partial fraction decomposition.
- Use partial fraction decomposition with linear factors to integrate rational functions.
- Use partial fraction decomposition with quadratic factors to integrate rational functions.
- Use long division with partial fraction decomposition to integrate rational functions

INTEGRATION BY TRIGONOMETRIC SUBSTITUTION

- Recognize the appropriate trigonometric substitution and relate that substitution to an appropriate right triangle.
- Know that integration by trigonometric substitution may require additional integration techniques.

INTEGRATION BY COMPLETING THE SQUARE

- Use the method of completing the square to find an antiderivative.

INTEGRATION OF TRIGONOMETRIC POWERS

- Use trigonometric identities to find antiderivatives of functions defined by trigonometric powers.

Learning Activities

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- EXPLORATION – unique challenges to study concepts as reinforcement and/or to study concepts not yet formally covered.
- SECTION PROJECTS – real life exercises solved by students using concepts and techniques generated in class.
- TECHNOLOGY – students use TI-89 graphing calculator and graphing software as tools to support and/or explore solutions graphically, analytically, and numerically.
- THINK PAIR SHARE – students work together by taking a moment to gather their thoughts and share them with their peers.

Assessments

- Graded homework for completeness and/or accuracy.
- Sectional quizzes.
- Sectional projects.
- Chapter or Mid-Chapter Tests.

21st Century Skills

X	Creativity	X	Critical Thinking	X	Communication X Collaboration
	Life & Career Skills		Information Literacy	X	Media Literacy

Interdisciplinary Connections

 Solve problems in physics, chemistry, and social sciences.

Technology Integration

8.1 Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.

- Daily use of Smartboard interface.
- Daily use of TI-89 graphing calculator.
- Weekly use of graphing/CAS software Winplot to enhance learning considering a geometrical (or graphical) approach.

Time Frame	**Block: 10 Days**

Topic

Parametric Equations & Polar Coordinates

Essential Questions

1. What is the intensity of radiation admitted from an antenna?
2. How far does a planet travel along its orbit?
3. How much area is needed to fill a parabolic shaped window bounded below by a circle?

Enduring Understandings

1. Represent curves or parts of curves parametrically.
2. Represent conic sections with rectangular and polar relations.
3. Understand the representation of area and arc length in rectangular and polar coordinates.

Alignment to NJSLS

SSE.B.3, CED.A.2, IF.A.1, IF.B.5

Key Concepts and Skills

PLANE CURVES AND PARAMETRIC EQUATIONS

- Sketch the graph of a curve given by a set of parametric equations.
- Eliminate the parameter in a set of parametric equations.
- Find a set of parametric equations to represent a curve

PARAMETRIC EQUATIONS AND CALCULUS

- Find the slope of a tangent line to a curve given by a set of parametric equations.
- Find the arc length of a curve given by a set of parametric equations. Find the area of a surface of revolution.

POLAR COORDINATES AND POLAR GRAPHS

- Understand the polar coordinate system.
- Rewrite rectangular coordinates and equations in polar form and vice versa.
- Sketch the graph of an equation given in polar form.
- Find the slope of a tangent line to a polar graph. Identify several types of special polar graphs.

AREA AND ARC LEGNTH IN POLAR COORDINATES

- Find the area of a region bounded by a polar graph.
- Find the points of intersection of two polar graphs.
- Find the arc length of a polar graph.
- Find the area of a surface of revolution.

POLAR EQUATIONS OF CONICS

- Analyze and write polar equations of conics.

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21st Century Skills

X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
	Life & Career Skills	X	Information Literacy		Media Literacy		

Interdisciplinary Connections

 Solving problems in science and architecture.

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DEPARTMENT: Mathematics COURSE: Multivariable Calculus

Time Frame	**Block: 10 Days**

Topic

VECTORS AND THE GEOMETRY OF SPACE

Essential Questions

1. How much work is needed to pull a sled up an incline?
2. How can we use vectors to analyze the flight of a plane?
3. How much torque is being applied to the fulcrum of a crank shaft?
4. How can we use vectors to write equations of plane?
5. How can the equations of functions in three dimensions be constructed visually?
6. How can we create equations for three dimensional space?
7. What are the defining characteristics for quadric surfaces and how are they determined?
8. What is the right hand rule for vectors and how does it apply in determining a solution?

Enduring Understandings

1. Represent position and motion in space using vectors.
2. Relation of vectors to their dot product and cross product.
3. Relation of two-space to three space via vectors.
4. Use of vectors to represent planes in space.

Alignment to NJSLS

N.VM.1, N.VM.2, N.VM.3, N.VM.4, N.VM.5, G.GMD.4, 8.2.12.E.1

Key Concepts and Skills

VECTORS IN THE PLANE

- Write the component form of a vector.
- Perform operations and interpret the results geometrically.
- Write a vector as a linear combination of standard unit vectors.

SPACE COORDINATES AND VECTORS IN SPACE

- Understand the three-dimensional rectangular coordinate system. Analyze vectors in space.

THE DOT PROCUCT OF TWO VECTORS

- Use properties of the dot product of two vectors.
- Find the angle between two vectors using the dot product.
- Find the direction cosines of a vector in space.
- Find the projection of a vector onto another vector.
- Use vectors to find the work done by a constant force.

THE CROSS PRODUCT OF TWO VECTORS IN SPACE

- Find the cross product of two vectors in space.
- Use the triple scalar product of three vectors in space.

LINES AND PLANES IN SPACE

- Write a set of parametric equations for a line in space.
- Write a linear equation to represent a plane in space.
- Sketch the plane given by a linear equation.
- Find the distance between points, lines, and planes in space.

SURFACES IN SPACE

- Recognize and write equations of cylindrical surfaces.
- Recognize and write equations of quadric surfaces.
- Recognize and write equations of surfaces of revolution.

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21st Century Skills

X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
	Life & Career Skills	X	Information Literacy		Media Literacy		

Interdisciplinary Connections

 Solving problems in science and architecture.

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Time Frame	**Block: 12 Days**

Topic

VECTOR-VALUED FUNCTIONS

Essential Questions

1. How can vector-valued functions be used to predict speed and velocity in space?
2. How can vector-valued functions be used in athletics?
3. How do we model three-dimensional structures using vector-valued functions?

Enduring Understandings

1. Extend the concepts of limits and continuity to vector-valued functions.
2. Analyze speed and velocity in space using vector-valued functions.
3. Determine the calculus of vector-valued functions.

Alignment to NJSLS

N.VM.3, G.C.5

Key Concepts and Skills

VECTOR-VALUED FUNCTIONS

- Analyze and sketch a space curve given by a vector-valued function.
- Extend the concepts of limits and continuity to vector-valued functions.

DIFFERENTIATION AND INTEGRATION OF VECTOR-VALUED FUNCTIONS

- Differentiate a vector-valued function. Integrate a vector-valued function.

VELOCITY AND ACCELERATION

- Describe the velocity and acceleration associated with a vector-valued function. Use a vector valued function to analyze projectile motion.

TANGENT VECTORS AND NORMAL VECTORS

- Find the unit tangent vector and a principal unit vector at a point on a space curve.
- Find the tangential and normal components of acceleration.

ARC LENGTH AND CURVATURE

- Find the arc length of a space curve.
- Use the arc length parameter to describe a plane curve of space curve.
- Find the curvature of a curve at a point on the curve. Use a vector-valued function to find frictional force.

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21st Century Skills

X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
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Time Frame	**Block: 20 Days**

DEPARTMENT: Mathematics COURSE: Multivariable Calculus

Topic

FUNCTIONS OF SEVERAL VARIABLES

Essential Questions

1. How can real-life problems be characterized by functions of several variables? Such as logging, business, and chemistry?
2. How do the concepts from single variable calculus apply and take form in multivariable calculus?
3. What is a function of several variables?
4. How do the concepts and rules of differentiation for functions of one variable extrapolate to functions of several variables?
5. How do you determine maximum, minimum and saddle points for functions of several variables?
6. What is the gradient of a function?
7. How does the method of Lagrange Multipliers work?

Enduring Understandings

1. Compute rates of change of functions of several variables.
2. Use gradient and normal vectors describe characteristics of curves.
3. Determine extrema of functions of several variables.
4. Lagrange multipliers as a method to solve boundary (constraint) problems.

Alignment to NJSLS

F.IF.2, F.IF.3, F.IF.4, F.IF.8, G.C.4, 8.2.12.E.1

Key Concepts and Skills

INTRODUCTION TO FUNCTIONS OF SEVERAL VARIABLES

- Understand the notation for a function of several variables.
- Read and interpret level curves and level surfaces.
- Use computer graphics to graph functions of two variables.

LIMITS AND CONTINUITY

- Understand the definition of a neighborhood in the plane.
- Understand the concept of continuity of functions of two and three variables.

PARTIAL DERIVATIVES

- Find and use partial derivatives of a function of two variables.
- Find and use partial derivatives of a function of three or more variables.
- Find higher-ordered partial derivatives of functions of two or three variables.

CHAIN RULES FOR FUNCTIONS OF SEVERAL VARIABLES

- Use the chain rule for functions of several variables.
- Find partial derivatives implicitly.

DIRECTIONAL DERIVATIVES AND GRADIENTS

- Find and use directional derivatives of a function of two and three variables.
- Find the gradient of a function of two variables.
- Use the gradient of a function of two variables in applications.

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Time Frame	**Block: 14 Days**

Topic

MULTIPLE INTEGRATION

Essential Questions

1. How can real-life problems be described by multiple integrals? Such as average production of automobiles, population density, and center pressure on a sail.
2. How do the concepts of the definite integral relate to multiple integrals in multivariable calculus?
3. What is the definition of a multiple integral?
4. How can an integral from Cartesian coordinates be converted to Polar, Cylindrical or Spherical coordinates?
5. How are multiple and iterated integrals applied to real world problems?

Enduring Understandings

1. Represent area and volume with multiple integrals.
2. Change variables of a multiple integral.
3. Represent and evaluate multiple integrals that determine the center of mass.
4. Lagrange multipliers as a method to solve boundary (constraint) problems.

Alignment to NJSLS

N.CN.4, G.GMD.4, A.CED.2, A.SSE.2, F.IF.8, 8.2.12.E.1

Key Concepts and Skills

INTERATED INTEGRALS AND AREA IN THE PLANE

- Evaluate an iterated integral.
- Use an iterated integral to find the area of a plane region.

DOUBLE INTEGRALS AND VOLUME
- Use a double integral to represent volume of a solid and use properties of double integrals.
- Evaluate a double integral as an iterated integral.
- Find the average value of a function over a region.

CHANGE OF VARIABLES: POLAR COORDINATES
- Write and evaluate double integrals in polar coordinates.

CENTER OF MASS AND MOMENTS OF INERTIA
- Find the mass of a planar lamina using a double integral.
- Find the center of mass of a planar lamina using double integrals.
- Find moments of inertia using double integrals.

SURFACE AREA
- Use a double integral to find the area of a surface.

TRIPLE INTEGRATION AND APPLICATIONS
- Use triple integrals to find volume of a solid region.
- Find the center of mass and moments of inertia of a solid region.

TRIPLE INTEGRATION IN OTHER COORDINATES
- Write and evaluate triple integrals in cylindrical coordinates.
- Write and evaluate triple integrals in spherical coordinates.

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Interdisciplinary Connections

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Time Frame	**Block: 14 Days**

Topic

VECTOR ANALYSIS

Essential Questions

1. How are vector fields determined to be conservative? What does this mean?
2. How can a line integral be used to find the mass of a spring?
3. How can curl be used to analyze the motion of a rotating liquid?
4. What is a vector field?
5. What is the fundamental theorem for line-integrals?
6. What is Green's Theorem?
7. What is the relationship between curl and divergence?
8. What is the relationship between the Fundamental Theorem of Calculus, Fundamental Theorem for Line Integrals, Green's Theorem, and Stoke's Theorem and the Divergence Theorem?

Enduring Understandings

1. Different applications of vector fields.
2. Determining the conservation of a vector field.
3. Meaning of the theorems of Green and Stoke.

Alignment to NJSLS

F.LE.5, N.VM.1, N.VM.5.b, N.VM.6, N.VM.11

Key Concepts and Skills

VECTOR FIELDS

- Understand the concept of a vector field.
- Determine whether a vector field is conservative.
- Find the curl of a vector.
- Find the divergence of a vector field.

LINE INTEGRALS

- Understand and use the concept of a piecewise smooth curve.
- Write and evaluate a line integral.
- Write and evaluate a line integral of a vector field.
- Write and evaluate a line integral in differential form.

CONSERVATIVE VECTOR FIELDS AND INDEPENDENCE OF PATH

- Understand the use of the Fundamental Theorem of Line Integrals.
- Understand the concept of independence of path.
- Understand the concept of conservation of energy.

GREEN'S THEOREM

- Use Green's Theorem to evaluate a line integral. Use alternative form of Green's Theorem.

PARAMETRIC SURFACES

- Understand the definition of a parametric surface.
- Find a set of parametric equations to represent a surface.
- Find a normal vector and tangent plane to a parametric surface.
- Find the area of a parametric surface.

SURFACE INTEGRALS

- Evaluate a surface integral as a double integral.

- Evaluate a surface integral for a parametric surface.
- Determine the orientation of a surface.
- Understand the concept of a flux integral.

DIVERGENCE THEOREM

- Understand and use the Divergence Theorem.
- Use the Divergence Theorem to calculate flux

STOKE'S THEOREM

- Understand and use Stoke's Theorem.

Use curl to analyze the motion of a rotating liquid.

Learning Activities

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