

## Board Approved August 2017

DEPARTMENT: MathematicsCOURSE: Precalculus

Week	Marking Period 1	Week	Marking Period 3
1	Functions	19	Angles & Their Measures- Unit Circle
2	Functions	20	Angles & Their Measures- Unit Circle
3	Functions	21	Angles & Their Measures- Unit Circle
4	Operations with Functions	22	Angles & Their Measures- Unit Circle
5	Polynomial & Rational Functions	23	Angles & Their Measures- Unit Circle
6	Polynomial & Rational Functions	24	Trig Graphs
7	Polynomial & Rational Functions	25	Trig Graphs
8	Polynomial & Rational Functions	26	Trig Graphs
9	Polynomial & Rational Functions	27	Trig Graphs
Week	Marking Period 2	Week	Marking Period 4
10	Polynomial & Rational Functions	28	Solving Trig Equations & Identities
11	Polynomial & Rational Functions	29	Solving Trig Equations & Identities
12	Polynomial & Rational Functions	30	Solving Trig Equations & Identities
13	Exponential & Logarithmic Functions	31	Solving Trig Equations & Identities
14	Exponential & Logarithmic Functions	32	Analyzing Triangles
15	Exponential & Logarithmic Functions	33	Analyzing Triangles
16	Exponential & Logarithmic Functions	34	Analyzing Triangles
17	Angles & Their Measures- Unit Circle	35	Intro to Limits & Continuity
18	Angles & Their Measures- Unit Circle	36	Intro to Limits & Continuity

Precalculus

<b>Time Frame</b>	<b>Standard 16 days</b>	<b>Block 8 days</b>					
<b>Topic</b>							
<b>Functions</b>							
<b>Essential Questions</b>							
<ol style="list-style-type: none"> <li>How are functions and their graphs related?</li> <li>How can technology be used to investigate properties of family of functions and their graphs?</li> <li>What are some patterns in the manipulation or changes in functions?</li> </ol>							
<b>Enduring Understandings</b>							
<ol style="list-style-type: none"> <li>Graphs and equations are alternative ways for depicting and analyzing patterns of non-linear change.</li> <li>Mathematical models can be used to describe physical relationships; these are often non-linear.</li> <li>Creating a graph is not the same as interpreting the information displayed.</li> <li>A variety of families of functions and methods can be used to model and solve real world situations.</li> </ol>							
<b>Alignment to NJSL</b>							
<b>F-IF.1, F-IF.4, F-IF.7, F-IF.7b, F-BF.3</b>							
<b>Key Concepts and Skills</b>							
<ol style="list-style-type: none"> <li>Determine if a relationship represents a function.</li> <li>Determine the inverse of a function and find the domain.</li> <li>Determine 1-1 functions.</li> <li>Determine whether a function is odd, even or neither – algebraically or graphically.</li> <li>Graph the basic parent functions; including linear, absolute value, quadratic, square root, cube root, exponential, logarithmic and reciprocal with and without the use of technology.</li> <li>Identify the key characteristic of the parent functions using domain, range, local maxima and minima, global maxima and minima, and intervals of increasing and decreasing.</li> <li>Write absolute value functions as piecewise functions.</li> <li>Graph piecewise functions and describe domain and range.</li> <li>Identify key characteristics of a quadratic function; including vertex intercepts, and axis of symmetry, using both algebraic and graphical approaches.</li> <li>Solve real-world problems involving a variety of functions.</li> <li>Transform functions.</li> </ol>							
<b>Learning Activities</b>							
CBL Activities – Matching the Graph, Time v. Distance, Modeling Step Functions. Scatter Plots – Creating <i>Best Fit Lines</i> ; analyze goodness of fit, use for prediction. Graphing Calculator Discovery – Characteristics of Family of Curves, Effects of Transformations.							
<b>Assessments</b>							
Common Quizzes & Tests Homework, Classwork Journal Writing & Portfolio Creation							
<b>21<sup>st</sup> Century Skills</b>							
x	Creativity	x	Critical Thinking	x	Communication	x	Collaboration
x	Life & Career Skills	x	Information Literacy		Media Literacy		
<b>Interdisciplinary Connections</b>							
History & Science – Regression Models used for prediction and analysis. Physics – Modeling Projectile Motion							
<b>Technology Integration</b>							

Precalculus

8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.

TI-83/84/89 Graphing Calculator, CBL/CBR, Responders

<b>Time Frame</b>	<b>Standard 4 days</b>	<b>Block 2 days</b>					
<b>Topic</b>							
<b>Operations of Functions</b>							
<b>Essential Questions</b>							
<ol style="list-style-type: none"> <li>How are functions and their graphs related?</li> <li>How can technology be used to investigate properties of families of functions and their graphs?</li> <li>How does explaining a process help me to better understand the idea?</li> <li>What are some patterns in the manipulation or changes in function?</li> </ol>							
<b>Enduring Understandings</b>							
<ol style="list-style-type: none"> <li>Function families have common characteristics.</li> <li>We can use Algebra to help graph functions.</li> <li>Technology is a resource for discoveries and investigations.</li> <li>There is a difference between using technology to find the answer compared to using technology to aid in supporting the answer.</li> </ol>							
<b>Alignment to NJSLS</b>							
<b>A-APR.6</b>							
<b>Key Concepts and Skills</b>							
<ol style="list-style-type: none"> <li>Form the sum, difference, product, and quotient of two functions, both polynomial and rational, and find its domain.</li> <li>Form the composite function and find the domain.</li> <li>Find the components of a composite function.</li> </ol>							
<b>Learning Activities</b>							
Have students discover multiple compositions and decompositions for a given function.							
<b>Assessments</b>							
Quizzes and Common Chapter Tests Homework and Classwork Journal writing and portfolios							
<b>21<sup>st</sup> Century Skills</b>							
X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
X	Life & Career Skills	X	Information Literacy	X	Media Literacy		
<b>Interdisciplinary Connections</b>							
Business – using different business models together, given specific parameters. Science – temperature conversion							
<b>Technology Integration</b>							
8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge. TI-83/84/89 Graphing Calculators responders							

<b>Time Frame</b>	<b>Standard 38 days</b>	<b>Block 19 days</b>
<b>Topic</b>		
<b>Polynomial and Rational Functions</b>		
<b>Essential Questions</b>		

## Precalculus

1. How do the characteristics of graphs relate to their corresponding equations?
2. How can algebra help us get information about a graph from an equation?
3. What are common characteristics of polynomials?
4. What is the best method for graphing polynomials?
5. How can technology assist in the understanding of graphing polynomials?

### Enduring Understandings

1. Functions can be written in several ways – certain forms can be helpful under different circumstances.
2. Families of functions have common characteristics.
3. We can use algebra to help graph functions.
4. The vocabulary is just as important as the math.

### Alignment to NJSLS

**N-CN.8, N-CN.9, A-APR.2, A-APR.3, A-APR.6, F-IF.7, F-IF.7c**

### Key Concepts and Skills

1. Examine graphs to determine end behavior.
2. Identify polynomial functions and find the degree given both factored and standard form.
3. Identify the zeros of a polynomial functions and its multiplicity.
4. Investigate the role of multiplicity with respect to the graph of a polynomial.
5. Investigate the role of the degree of a polynomial with respect to the number of turning points.
6. Analyze the graph of a polynomial function given both factored and standard form; identify the intercepts, end behavior, and turning points.
7. Form polynomials from zeros and graphs.
8. Use a graphing utility to approximate values of turning points.
9. Factor polynomials of higher degree.
10. Divide polynomials with long division and synthetic division.
11. Apply the remainder and factor theorems to find remainders and factors.
12. Apply the rational zero theorem to the list of potential zeros of a polynomial.
13. Find the real and complex zeros of a polynomial.
14. Apply the conjugate pair theorem to help write a polynomial in standard form and to help find other zeros.
15. Form a polynomial with specified zeros, both real and complex.
16. Solve polynomial equations for both real and complex zeros.
17. Find the domain of a rational function.
18. Determine the vertical asymptotes of a rational function.
19. Determine the horizontal and oblique asymptotes of a rational function.
20. Analyze the graph of rational function; determining the domain, intercepts symmetry, vertical, horizontal, and oblique asymptotes, and end behavior.
21. Sketch the graph of a rational function by hand.
22. Solve polynomial or rational inequalities algebraically and graphically (Advanced & Honors only).
23. Decompose a rational expression into partial fractions (Honors Only).

### Learning Activities

Model Concentration of Medicine in Blood Stream.

Build Polynomials from given characteristics.

CBL – Investigating Boyle’s Law.

Construct Graphs given limited information.

Investigation of End Behavior of  $n^{\text{th}}$ -degree Polynomials with positive and negative coefficients.

### Assessments

## Precalculus

Quizzes & Common Chapter Tests, Homework and Classwork, Journal Writing and Portfolios							
<b>21<sup>st</sup> Century Skills</b>							
X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
X	Life & Career Skills	X	Information Literacy	X	Media Literacy		
<b>Interdisciplinary Connections</b>							
Business – Manufacturing Predictions History – Population Modeling Science – Boyle’s Law: Relating Volume, Temperature, and Pressure, Light Intensity.							
<b>Technology Integration</b>							
8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge. TI-83/84/89, Responders CBL							

<b>Time Frame</b>	<b>Standard 16 days</b>	<b>Block 8 days</b>
<b>Topic</b>		
<b>Exponential and Logarithmic Functions</b>		
<b>Essential Questions</b>		
<ol style="list-style-type: none"> <li>5. How can an exponential function represent a real-world scenario?</li> <li>6. How can the properties of logarithms be used to solve equations?</li> <li>7. Why does simplifying or expanding a logarithmic expression help us solve problems?</li> <li>8. Why is the number <math>e</math> important?</li> </ol>		
<b>Enduring Understandings</b>		
<ol style="list-style-type: none"> <li>5. Functions families have common characteristics.</li> <li>6. We can use algebra to help graph functions.</li> <li>7. Knowing how to read an equation is essential for graphing the function.</li> <li>8. Mathematics applies to the sciences.</li> </ol>		
<b>Alignment to NJSL</b>		
<b>F-IF-7, F-IF.7e, F-LE.4</b>		
<b>Key Concepts and Skills</b>		
<ol style="list-style-type: none"> <li>4. Evaluate exponential functions.</li> <li>5. Graph exponential functions and transformations of such.</li> <li>6. Define the number <math>e</math>.</li> <li>7. Solve exponential equations.</li> <li>8. Change exponential expressions to logarithmic expressions and visa-versa.</li> <li>9. Evaluate logarithmic functions.</li> <li>10. Determine the domain of a logarithmic function.</li> <li>11. Graph logarithmic functions and transformations of such.</li> <li>12. Solve logarithmic equations.</li> <li>13. Expand and condense logarithmic expressions.</li> <li>14. Use properties of logarithms and exponents to solve equations algebraically.</li> <li>15. Use exponential and logarithmic functions to describe real world scenarios including growth and decay.</li> </ol>		
<b>Learning Activities</b>		
CBL – Newton’s Law of Cooling		

## Precalculus

Logistic – spread of a rumor/virus  
 Develop motivation for Logistic Model by conjecturing the conditions that population growth is limited.  
 M&M Lab – modeling exponential decay  
 Compound versus Simple Interest Activity

### Assessments

Quizzes and Common Chapter Tests  
 Homework and Classwork  
 Journal writing and portfolios

### 21<sup>st</sup> Century Skills

X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
X	Life & Career Skills	X	Information Literacy	X	Media Literacy		

### Interdisciplinary Connections

Science – population models (growth, decay, half-life, logistics models) Financial - investments

### Technology Integration

8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge. TI-83/84/89 Graphing Calculators responders

Time Frame	Standard 34 days	Block 17 days
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### Topic

### Angles and Their Measure – The Unit Circle

#### Essential Questions

1. What is Precalculus?
2. How does Precalculus prepare you for calculus?
3. Mathematically speaking, how does what you learned today connect with what you learned yesterday, and what you will learn tomorrow?
4. How can we find out information for all different types of triangles?

#### Enduring Understandings

1. Strong algebra skills are essential for success in Calculus.
2. Precalculus connects discrete pieces of mathematics from your past learning.
3. Trigonometry allows for indirect measurement of triangles when given limited information.
4. There exists a distinct and definite bridge between the worlds of algebra and algebraic thinking and the field of geometry.

#### Alignment to NJSL

**A-REI.4, F-TF.1, F-TF.2, G-SRT.8**

#### Key Concepts and Skills

1. Review, discuss, and practice algebra skills from summer assignment which include equations in one variable, linear equations in two variables, function notation, quadratic equations, synthetic substitution and division, radicals, rational expressions and domain.
2. Review, discuss, and practice trigonometry skills from summer assignment which include right triangle trigonometry and problems involving angles of elevation and depression.
3. Relate right triangle trigonometry to the ratio of sides of a triangle relating it to the algorithm used in a calculator.
4. Identify co-terminal and reference angles using degrees.
5. Create a unit circle from its relationship to right triangle trigonometry.
6. Use the unit circle to evaluate all six trigonometric relations; use the calculator to evaluate each relation.

Precalculus

7. Evaluate trigonometric functions of any angle using reference angles.

**Learning Activities**

CBL – Gathering temperatures at certain time intervals to determine period and amplitude.  
Angular/Linear Speed Relation – given one speed find the other and convert to appropriate units.

**Assessments**

Quizzes & Common Chapter Tests,  
Homework and classwork,  
Journal and Portfolio development.

**21<sup>st</sup> Century Skills**

X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
X	Life & Career Skills	X	Information Literacy		Media Literacy		

**Interdisciplinary Connections**

Science – Describing temperature on given intervals, modeling pendulum and spring motion.

**Technology Integration**

8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.

TI-83/84/89, CBL

<b>Time Frame</b>	<b>Standard 18 days</b>	<b>Block 9 days</b>
<b>Topic</b>		
<b>Trigonometric Graphs</b>		
<b>Essential Questions</b>		
	<ol style="list-style-type: none"> <li>How do we use the unit circle to create a graph for each of the six trigonometric functions?</li> <li>Where do the common characteristics of sinusoidal functions come from?</li> <li>How does the equation for a function affect the graph of that function?</li> <li>What does it mean to find an exact value?</li> </ol>	
<b>Enduring Understandings</b>		
	<ol style="list-style-type: none"> <li>The unit circle is a gateway between geometry and algebra.</li> <li>Members within a family of functions have common characteristics.</li> <li>Inverse functions can be found and verified in a variety of ways.</li> <li>Decimals can be good approximations of exact values.</li> </ol>	
<b>Alignment to NJSL</b>		
<b>F-IF.7, F-IF.7e, F-TF.5</b>		
<b>Key Concepts and Skills</b>		
	<ol style="list-style-type: none"> <li>Graph sine and cosine functions applying the knowledge of the unit circle.</li> <li>Sketch the graph of tangent and cotangent functions.</li> <li>Sketch the graph of secant and cosecant functions using the graphs of cosine and sine as aids.</li> <li>Describe important characteristics of the graphs of the six trigonometric functions (domain, range, max/min, increasing, decreasing, x-intercepts, y-intercepts).</li> <li>Sketch the graphs of the sine and cosine functions both by hand and with graphing technology using the concepts of amplitude and period including those with transformations.</li> <li>Write equations for transformed sine and cosine functions.</li> <li>Determine the restrictions on the domain for the sine functions and the cosine function in order for the inverse to also be a function.</li> <li>Calculate both exact and approximate values for inverse sine and cosine.</li> </ol>	
<b>Learning Activities</b>		
CBL – Tuning Fork Lab (sinusoid)		

## Precalculus

Discover applications of sinusoids (tides, Ferris Wheel, average temp (temp versus month), harmonic motion, etc)

### Assessments

Quizzes and Common Chapter Tests  
Homework and Classwork  
Journal writing and portfolios

### 21<sup>st</sup> Century Skills

X	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
X	Life & Career Skills	X	Information Literacy		Media Literacy		

### Interdisciplinary Connections

Science – Tide charts, temperature fluctuations.

### Technology Integration

8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge. TI-83/84/89 Graphing Calculators responders

Time Frame	Standard 20 days	Block 10 days					
<b>Topic</b>							
<b>Solving Trigonometric Equations and Identities</b>							
<b>Essential Questions</b>							
<ol style="list-style-type: none"> <li>1. What does it mean when we solve a trigonometric equation?</li> <li>2. How do we represent multiple solutions on a given domain?</li> <li>3. What does it mean to prove an identity?</li> <li>4. How can we use identities to simplify the process involved in solving an equation?</li> </ol>							
<b>Enduring Understandings</b>							
<ol style="list-style-type: none"> <li>1. The solutions to an equation can be represented graphically as the intersection of two functions.</li> <li>2. Sinusoidal functions have an infinite number of solutions that are the same interval apart.</li> <li>3. Equivalent expressions can be written in a variety of formats.</li> <li>4. There exists a distinct and definite connection between the fields of geometry and algebra.</li> </ol>							
<b>Alignment to NJSL</b>							
<b>F-TF.8</b>							
<b>Key Concepts and Skills</b>							
<ol style="list-style-type: none"> <li>1. Solve equations involving sine and cosine using algebraic and graphical techniques.</li> <li>2. Solving equations involving frequency changes of sine and cosine using algebraic and graphical techniques.</li> <li>3. Find exact values of trigonometric functions using sum and difference formulas.</li> <li>4. Find exact values of trigonometric functions using double angle and half angle identities.</li> <li>5. Prove trigonometric identities using Pythagorean, reciprocal, and quotient identities.</li> </ol>							
<b>Learning Activities</b>							
Connections to Algebra – Relate familiar factoring skills to trigonometric relations to solve trigonometric equations. Verifying Trigonometric Identities – A Graphical Approach Solving Trigonometric Equations – A Graphical Approach							
<b>Assessments</b>							
Quizzes & Common Chapter Tests, Homework and classwork, Journals and Portfolio Development							
<b>21<sup>st</sup> Century Skills</b>							
	Creativity		Critical Thinking		Communication		Collaboration

Precalculus

Life & Career Skills	Information Literacy	Media Literacy
<b>Interdisciplinary Connections</b>		
Science – Modeling wave motion, pendulum motion, and predicting temperatures.		
<b>Technology Integration</b>		
8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge.		
TI-83/84/89, CBL, Interactive WhiteBoard		

<b>Time Frame</b>	<b>Standard 16 days</b>	<b>Block 8 days</b>					
<b>Topic</b>							
<b>Analyzing Triangles</b>							
<b>Essential Questions</b>							
<ol style="list-style-type: none"> <li>How do we use algebra to manipulate expressions to do what we want them to do?</li> <li>Why does the law of sines have an ambiguous case?</li> <li>How can we use trigonometry to solve any type of triangle?</li> <li>How can representing physics applications using vectors help us determine the resultant effect of multiple forces on an object?</li> </ol>							
<b>Enduring Understandings</b>							
<ol style="list-style-type: none"> <li>There exists a definite and distinct bridge between the fields of trigonometry and physics.</li> <li>Trigonometry can be used to indirectly measure oblique as well as right triangles.</li> <li>Characteristics of sinusoidal functions can affect how we use them to solve right triangles.</li> <li>Multiple ways of representing real world phenomena can be used to solve problems.</li> </ol>							
<b>Alignment to NJSL</b>							
<b>G-SRT.11</b>							
<b>Key Concepts and Skills</b>							
<ol style="list-style-type: none"> <li>Use the law of sines to solve oblique triangles.</li> <li>Find the area of oblique triangles using the sine function.</li> <li>Use the law of cosines to solve oblique triangles.</li> <li>Use Heron’s Formula to find the area of a triangle.</li> <li>Represent vectors as a directed line segment.</li> <li>Write vectors in component form, perform basic vector operations with vectors, and find the direction angle of a vector.</li> <li>Find the resultant of two vectors.</li> <li>Use vectors to solve real life problems.</li> <li>Find the dot product of two vectors and find the angle between two vectors.</li> </ol>							
<b>Learning Activities</b>							
Derivation of Law of Sines & Cosines and area of a triangle involving sine. Applications of Right Triangle Trigonometry, Law of Sines & Cosines to determine heights of objects on flat and non-flat surfaces.							
<b>Assessments</b>							
Common Quizzes & Tests Homework, Classwork Journal Writing & Portfolio Creation							
<b>21<sup>st</sup> Century Skills</b>							
x	Creativity	x	Critical Thinking	x	Communication	x	Collaboration
x	Life & Career Skills	x	Information Literacy		Media Literacy		
<b>Interdisciplinary Connections</b>							
Science – Describing motion in component form, computing resultant vectors.							

Precalculus

<b>Technology Integration</b>							
8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge. TI-83/84/89 Graphing Calculator, CBL/CBR, Responders							
Time Frame	Standard 8 days			Block 4 days			
<b>Topic</b>							
<b>Introduction to Limits and Continuity (Advanced and Honors Only)</b>							
<b>Essential Questions</b>							
<ol style="list-style-type: none"> <li>1. What characteristics does a function need to have for a limit to exist?</li> <li>2. What does it mean to have a limit equal infinity</li> <li>3. Why does the slope of a secant line change as <math>x</math> approaches a point?</li> <li>4. What is rate of change and how can functions and graphs help model it?</li> <li>5. Is a function continuous at a particular value of <math>x</math>?</li> <li>6. Is a function considered a continuous function?</li> </ol>							
<b>Enduring Understandings</b>							
<ol style="list-style-type: none"> <li>1. There are different ways to evaluate limits.</li> <li>2. Multiple ways of representing phenomena can be helpful in the evaluation process.</li> <li>3. The average rate of change between two points on a function can be written as a function.</li> <li>4. Functions can be used to represent a variety of phenomena.</li> <li>5. To understand the formal definition of continuity at a particular value of <math>x</math>.</li> </ol>							
<b>Alignment to NJSL</b>							
<b>F-IF.6</b>							
<b>Key Concepts and Skills</b>							
<ol style="list-style-type: none"> <li>1. Define a limit.</li> <li>2. Find the value of a limit graphically and as a table, including limits at infinity.</li> <li>3. Find the limit algebraically</li> <li>4. Find one sided limits graphically and algebraically.</li> <li>5. Determine the <math>x</math>-values at which a function is continuous/discontinuous</li> <li>6. Find the average rate of change of a secant line (Honor only) for a function.</li> <li>7. Find the slope of a line tangent to a function at a given point using limits (Honors only)</li> <li>8. Find limits and determine continuity of piecewise functions.</li> </ol>							
<b>Learning Activities</b>							
Graph piecewise functions by hand. Use graphs to determine, one-sided limits and continuity of those functions.							
<b>Assessments</b>							
Quizzes and Common Chapter Tests Homework and Classwork Journal writing and portfolios							
<b>21<sup>st</sup> Century Skills</b>							
	Creativity	X	Critical Thinking	X	Communication	X	Collaboration
X	Life & Career Skills	X	Information Literacy		Media Literacy		
<b>Interdisciplinary Connections</b>							
Science – limiting values of populations.							
<b>Technology Integration</b>							
8.1 Educational Technology- All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively and to create and communicate knowledge. TI-83/84/89 Graphing Calculators responders							